

these methods the Russians have made ample provision, so have the American astronomers, and the Germans will occupy at least one station, Tchefoo, specially for these methods. Every preparation is being made, in fact, for Northern work (except only that our North Indian region, available for these methods as well as Delisle's, is not sufficiently provided for). But now what is there to balance all this, in the Southern hemisphere? *Of really first-class stations there are but three which have even been mentioned,—viz., Crozet Island, Macdonald Island, and Kerguelen Island. Of these only Kerguelen Island has been actually selected; and here bad weather is almost a certainty. Of the other stations Canterbury (N.Z.), Chatham Island, Bourbon, Mauritius, and Rodriguez, it is only necessary to remark that they are very inferior for these three important methods.*

It is on this account chiefly that I have been earnest in my appeal for the occupation of Antarctic and sub-Antarctic stations. If anything were required to add to my anxiety on this subject, it would be found in the manifest reliance placed by Russia, America, and Germany, on the methods in question.

I am concerned to think that reconnaissances over the regions between Kerguelen Island, Enderby Land, Possession Island, and Auckland Island, may be absolutely necessary for a proper choice of stations; that such reconnaissances might have been made since I first dwelt on these matters four years ago; and that possibly had I been earnest in advocating these considerations during the last four years, either Great Britain or America might before this have found suitable observing stations in the above named region. I judged it best simply to indicate the state of the case and wait. I fear I may have been mistaken, though it is difficult to see what could have been done until the approach of the event itself and the declared intentions of other countries enforced attention to the circumstances I have touched upon. I trust it may still not be too late to provide for an adequate number of Southern stations sufficiently far apart to give proper chances of success. I do not hesitate to say that in my opinion the provision hitherto made is altogether inadequate, so far as Southern stations are concerned.

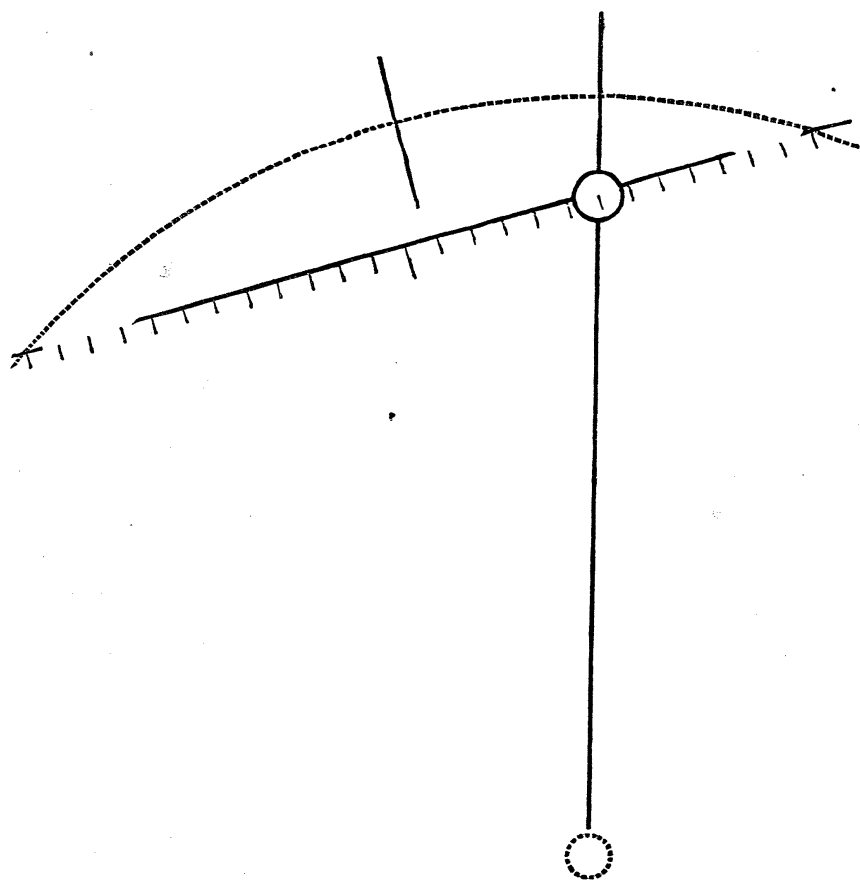
A Graphical Representation of the Circumstances of the Transit of Venus in 1874, with a view to show the sufficiency of this method for the purposes of Prediction. By F. C. Penrose, Esq.

Notwithstanding the admirable diagrams which Mr. Proctor has laid before the Society, it may still be interesting that the subject should be looked at in various ways. I can hardly suppose but that Mr. Proctor, considering the great attention he

has paid to the matter, may have anticipated me in this, but the subject is so important that some repetition may be excused.

The method I have used is the same, to all intents and purposes, as that which I have used in predicting occultations and eclipses. Indeed, until dealing with the refinements which will be necessary in discussing observations when we get them, it is rather a simpler case than an eclipse or a planetary occultation.

The woodcut is a perspective projection of the phenomenon. The plane of the projection is at right angles to the Earth's radius vector at the moment of conjunction in R. A. of \odot and φ . The distance is the radius vector of φ , and the point of



sight (as it is called), and which here is the same as the projecting point, is the centre of the Sun. The dotted line is the Sun's limb, the inclined line is the path of φ . The vertical line is the line of conjunction in R.A. The circle there represented is the true size of *Venus* in scale with the Sun; the dotted circle below is the Earth as projected on to the plane of the picture, and therefore obviously measuring $P - p$, where P and p are the horizontal parallaxes of *Venus* and Sun respectively.

This diagram suffices for all the purposes of prediction ; in

proof of this, I submit certain results obtained from it with the calculations given in the *Nautical Almanac*.

Internal contact considered in every case. The times mentioned are local mean time.

		Graphic Method.		Nautical Almanac.	
		h	m	h	m
Alexandria: Egress..	..	20	7 ^o 0	20	7 ^o 0
Kerguelen: Ingress..	..	19	3 ^o 4	19	3 ^o 0
Egress	22	28 ^o 7	22	28 ^o 7
Auckland, New Zealand: Egress		5	27 ^o 7	5	27 ^o 5
Honolulu: Ingress	3	33 ^o 3	3	33 ^o 0

Least Distance of Centres.

16^h 6^m 35^s G.M.T. against 16^h 6^m 32^s.

I have no doubt—indeed I am sure—that the graphic process is easily susceptible of still greater accuracy, and this, I think, confirms the high expectations formed by Dr. De La Rue and others from photography.

The method is simply this: for any chosen place apply first roughly a radius measuring S—s, and see to a few minutes on the planet's time path (which can be done at once) where it cuts that line, and what is about the time before or after conjunction. For instance, let us take Honolulu, Latitude 21° 18' 24'', Longitude, 10^h 31^s 30^m. Then form the following equation:—

G.M.T. 2	h	m	s	Before	∠	h	m	s	h	m	s	Sun's R.A. corresponding.
	2	50	8			14	9	0	17	3	0	
Acceleration						0	2	20	10	31	30	Longitude <i>West</i> .*
S.T. at G. M. Noon						17	8	18	3	45	8	Hour Angle <i>East</i> .
						31	19	38	31	19	38	

The terrestrial parallax is now very easily found, either with the help of such a diagram as I have given in my work on Occultations, &c., or quite as readily by the solution by the aid of the slide rule of—

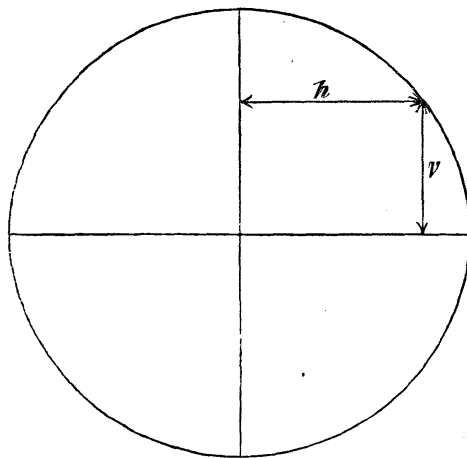
$$v = \sin \lambda \cos \delta + \cos \lambda \sin \delta \cos \text{hour-angle.}$$
$$h = \cos \lambda \sin \text{hour-angle.}$$

Such Southern stations are best which, like Kerguelen, see the ingress in the morning, and the egress towards noon; and for Northern Stations exactly the reverse holds good; as, for instance, Nertschinsk. So much so is this the case for the Southern stations that Kerguelen—latitude about 49°—has a course only one minute longer than Possession Island, so much to the south of it; whilst Enderby, of nearly the same latitude as Possession Island,

* If longitude is west it goes into the second column. If east into the first column. If the hour-angle which is required to balance the equation is found in the right-hand column it is east, otherwise west.

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would (supposing that a station there were practicable) give a course 4 minutes shorter than Kerguelen. The cause of this is easily seen. Suppose two places on the same parallel of latitude, but of such difference of longitude that one should see the ingress at noon and the egress in the afternoon, whilst the other sees the ingress in the morning and the egress at noon. It is obvious by looking at the diagram that the path of the first is very oblique to the direction of the radius $S-s$, whilst the other is much more nearly in the same direction, it consequently produces much more effect in practically reducing this radius, and cutting off a shorter segment from the time-path. This is in favour of a Southern station. At a Northern station, on the contrary, where a lengthened path is desired, such positions as produce the least possible reduction of the radius $S-s$, and consequently the reverse condition of longitude, are most desirable. This consideration explains the great value of Nertschinsk, in addition to its high Northern latitude; but there are places in Japan, and especially on the coast of the mainland near Lake Kinka, which are nearly as good, and might very likely be better stations for observation. A similarly constructed diagram for the transit of 1882 (the elements of which have been kindly communicated to me by Mr. Hind) has enabled me to confirm all that Mr. Proctor has said about the great superiority of the transit of 1874 to that of 1882. The utmost result which could be got for Delisle's method in that year would be under 17 minutes. In 1874 the combination of Kerguelen and Honolulu gives 23 minutes; and if Crozet's Island could also be occupied, it would be very greatly to be desired, because, although not quite so southerly as Kerguelen, the consideration of the path which I have mentioned acts more in its favour, and the time-path would be about six-tenths of a minute more favourable, either for Delisle's or Halley's process. The altitude at ingress is perhaps not quite high enough, but still 13° . Everything, at any rate, seems in favour of straining a point to utilise the transit of 1874 as much



as possible. Indeed I am inclined to expect that the best results in 1882 will be from photography, especially with the help of the experience of the coming year. I am not competent, nor do I venture, to offer any opinion whether Halley's or Delisle's method is the best; but when so little is asked as to do our best to make sure both of Kerguelen and Crozet's Island, and to do something to support Nertschinsk in or near Japan, for the sake of

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having two strings to our bow; it is at any rate worth urgently raising the question, and speaking before it is too late. On one point my graphical investigation has led me to a consideration which at first sight, at any rate, seems to be in favour of Delisle's method as against Halley's, and it is this: if there is any error in the declination of *Venus*, or the measure of the semidiameters, it would entail a greater error on Halley's than on Delisle's method. As far as I have thus far made out, an error of 1^s in these measures would affect the time-path to the extent of about 9^s for Halley's method, and about 5^s for Delisle's; but this is perhaps only an additional reason why the two methods should be resorted to, if only for the sake of clearing up any such possible error, or errors.

April 9, 1873.

Graphical Method for Determining the Motion of a Body in an Elliptic Orbit under Gravity. By Richard A. Proctor, B.A., Cambridge.

The student of astronomy often has occasion to determine approximately the motion of bodies, as double stars, comets, meteor systems, and so on,—in orbits of considerable eccentricity; and therefore a graphical method for solving such problems in a simple yet accurate manner will probably be of use to many readers of the *Notices*. The process now to be described is, so far as I am aware, a new one, (though I have an indistinct recollection of a paper, by Mr. Waterston, if I remember rightly, suggesting that part of the construction which relates to the curve of sines).^{*} Of course it involves no new principles. By its means a figure, such as the lithographed diagram illustrating this paper, having once for all been carefully inked in on good drawing card, the motion of a body in an orbit of any eccentricity can be determined by a pencilled construction of great simplicity, which can be completed (including the construction of the ellipse) in a second or two.

Let $AP A'$ be an elliptical orbit of which $AC A'$ is the major axis, S being the centre of force, so that A is the aphelion, and A' the perihelion. Let a be the half major axis; e the eccentricity CS ; H half the periodic time, and T the time in which the body moves from A to P .

On AA' describe the auxiliary semicircle $A b A'$.

Then

$$\begin{aligned} T &\propto \text{area } ASP \propto \text{area } ASQ \\ &\propto \text{area } (ACQ + SCQ) \\ &\propto a(AQ) + e(QM) \\ &\propto AQ + \frac{e}{a} \cdot QM. \quad (a) \end{aligned}$$

* See addendum to present paper.